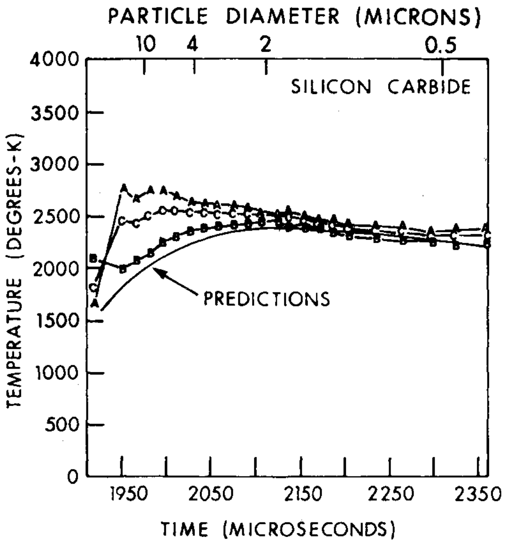

Fig. 4 Temperature measurements and predictions for station 2.

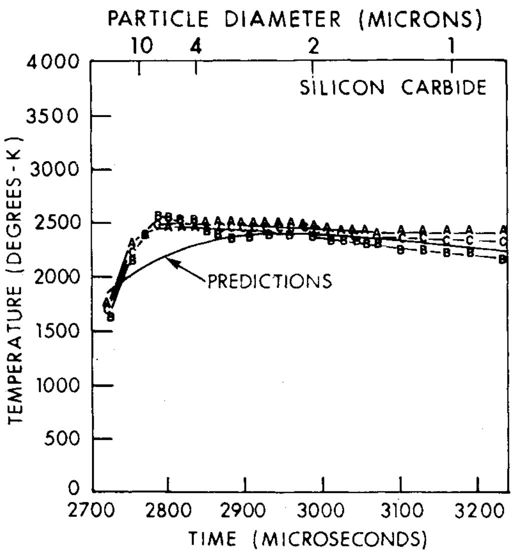

Fig. 5 Temperature measurements and predictions for station 3.

sphere for the actual test conditions. An x-ray photograph was used to obtain the initial launch velocity in the chamber.

Figures 2, 4, and 5 show the actual temperature computed using the observed radiometric data at the three stations. The time scale has been estimated to be ± 10 μs, while the temperature scale could have an error margin of ± 100 K. Superimposed on these is a smoother black line giving predictions. One sees from these figures that the analytical scheme used works well. Since the temperature dependent emissivity was thought to be the most uncertain of all theoretical input, both tungsten and tungsten carbide were also analyzed using the same free-molecular derivations, which also gave good agreement with experimental observations.

Acknowledgment

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Correlations among Turbulent Shear Stress, Turbulent Kinetic Energy, and Axial Turbulence Intensity

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Nomenclature

a,b,A	= constants
d	= pipe radius, half channel width
D	= pipe diameter
f	= function
U_b	= mean or bulk velocity
U_m	= maximum velocity
U_τ	= shear velocity
u,v,w	= fluctuating turbulent velocity components in the x,y, and z, directions respectively
$\tilde{u},\tilde{v},\tilde{w}$	= root-mean-square values of u,v, and w, respectively
y	= distance from the wall
R	= Reynolds number for a pipe defined as $U_b D/\nu$
Re	= Reynolds number for a pipe defined as $U_m D/\nu$
Re_c	= Reynolds number for a channel defined as $U_m d/\nu$
$\tilde{u}^+,\tilde{v}^+,\tilde{w}^+$	= $\tilde{u}/U_\tau$, $\tilde{v}/U_\tau$, $\tilde{w}/U_\tau$, respectively
uv^+	= $\overline{uv}/U_\tau^2$
KE^+	= nondimensional turbulent kinetic energy = $\frac{1}{2}(\tilde{u}^{+2}+\tilde{v}^{+2}+\tilde{w}^{+2})$
y^+	= yU_τ/ν
ρ	= density
ν	= kinematic viscosity
τ_t	= turbulent shear stress = $-\overline{uv}\cdot\rho$
Subscripts	
b	= bulk
c	= channel
m	= maximum
τ	= shear
Superscripts	
$(\bar{\ })$	= time average of a quantity
$(\tilde{\ })$	= $[\overline{(\ )^2}]^{1/2}$, root mean square

Introduction

THE relationship between the turbulent shear stress and the turbulent kinetic energy has been examined in the past. Harsha and Lee[1] studied such a relationship and suggested the following linear equation for the boundary-layer, jet, and wake flows:

$$\tau_t = A\,\tfrac{1}{2}\,(\overline{u^2}+\overline{v^2}+\overline{w^2}) \tag{1}$$

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where A is a constant, which they gave as 0.3. Equation (1) may be written as

$$-\overline{uv}^+ = A(KE)^+ \qquad (2)$$

One disadvantage of Eq. (2) is that it is not satisfied at the pipe centerline, where the turbulent shear stress is zero but the turbulent kinetic energy is not. In the present study, the validity of Eq. (2) is examined further in relation to the following: 1) the inlet region of pipe and channel flows, 2) the inner layer of duct flow, and 3) the effect of surface roughness.

The relationship between the turbulence kinetic energy KE^+ and the axial component of turbulence intensity $\tilde{u}^+$ also is studied as a possible alternative that can be used in connection with various theoretical analyses applied in turbulence research. To carry out the investigation, the experimental data of Sandborn,[2] Laufer,[3,4] Lawn,[5,6] Clark,[7,8] and Comte-Bellot[9] are used. These authors have measured turbulence intensities and turbulent shear stress at various points in the flowfield of pipe and channel flows. The method of calculation used in this study is as follows: a number of points in the flowfield are selected, with y/d values ranging from 0.1 to 1. The turbulence intensities and turbulent shear stress at these points then are calculated from the data given in the preceding references. The quantities $\tilde{u}^+$, $\tilde{v}^+$, $\tilde{w}^+$, and $\overline{u\,v}^+$ are calculated at these points, from which the turbulent kinetic energy KE^+ is obtained. This allows a plot of $\overline{uv}^+$ against KE^+ (or KE^+ against $\tilde{u}^+$, as required). To investigate the variation of $\overline{uv}^+$ with KE^+ and KE^+ with $\tilde{u}^+$ in the inner layer, a number of data points are selected close to the wall with $y/d<0.1$, the smallest value of y^+ obtained being 5.

Results and Discussion

Validity of Eq. (2) to Pipe and Channel Flows

The relationship between $\overline{uv}^+$ and KE^+ for turbulent pipe flow for points having y/d values ranging from 0.1 to 1 is shown in Fig. 1a. The figure represents the data of Sandborn,[2] Laufer,[3] and Lawn[5] for fully developed, smooth pipe flow. In addition, the data of Lawn[5] for developing rough pipe flow ($x/d=27$) are presented. The plot indicates that Eq. (2) is not satisfied properly, with large scatter between the data being observed.

Figure 1b is a similar plot for smooth channel flow ($0.1 \leq y/d \leq 1$). The data of Laufer[4] and Clark[7] are for fully developed channel flow with an aspect ratio of 12, whereas those of Comte-Bellot[9] are for developing ($x/d=40$) as well as fully developed ($x/d=118$) channel flow with an aspect ratio of 13. It is clear from this figure that linearity between $\overline{uv}^+$ and KE^+ is not well established. Again, large scatter is observed between the data. Figure 1 suggests that the $\overline{u\,v}^+ - KE^+$ relationship is generally dependent on a number of factors, such as the Reynolds number, the axial location, and surface roughness.

In Fig. 2, the $\overline{uv}^+ - KE^+$ relationship is presented for the inner layer ($y^+<30$). Clark's data[7] are examined over a range of Reynolds numbers and y/d. It is seen that linearity between $\overline{uv}^+$ and KE^+ is not achieved. Furthermore, the $\overline{uv}^+ - KE^+$ relationship for the inner layer is seen to depend on the y^+ range considered, as well as the Reynolds number.

Relationship Between the Turbulent Kinetic Energy and the Axial Component of Turbulence Intensity

The data presented in Figs. 1 and 2 are replotted in the form $KE^+ = f(\tilde{u}^+)$ in Figs. 3 and 4, respectively. The latter figures suggest that the relationship between KE^+ and $\tilde{u}^+$ is effectively linear, i.e.,

$$KE^+ = a\tilde{u}^+ + b \qquad (3)$$

where a and b are constants.

Pipe flow data of Sandborn,[2] Laufer,[3] and Lawn,[5] over the range $0.1 \leq y/d \leq 1$, are shown in Fig. 3a. This figure illustrates that these data may be described by the equation

$$KE^+ = 2.24\tilde{u}^+ - 1.13 \qquad (4)$$

Equation (4) is applicable to rough pipes as well as to smooth ones, to the inlet and fully developed regions of pipe flow, and over a range of Reynolds numbers.

Channel flow data of Laufer,[4] Clark,[7] and Comte-Bellot[9] are represented in Fig. 3b. The data may be described by the

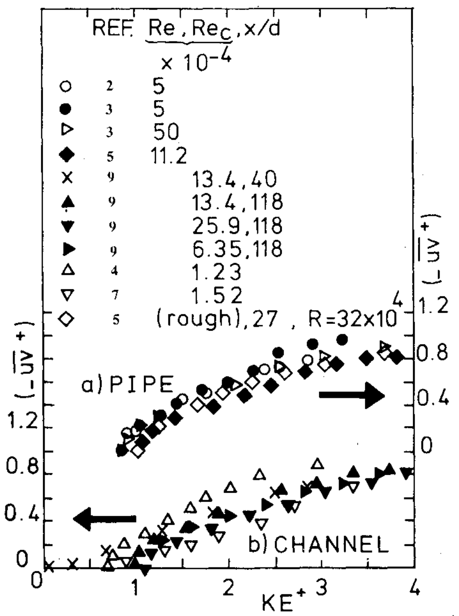

Fig. 1 Variation of $\overline{uv}^+$ with KE^+ for pipe and channel flows.

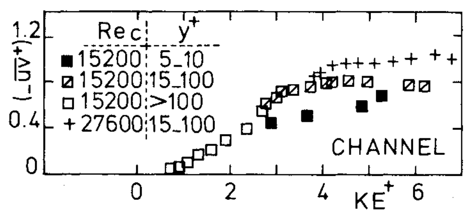

Fig. 2 Variation of $\overline{uv}^+$ with KE^+ in the inner layer.[7]

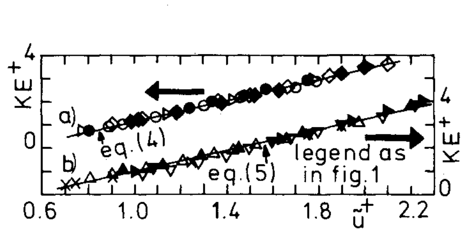

Fig. 3 Variation of KE^+ with $\tilde{u}^+$ for pipe and channel flows.

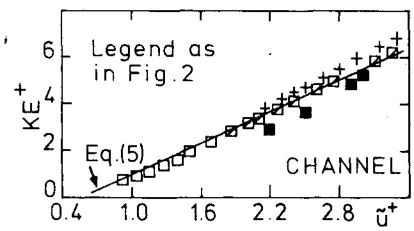

Fig. 4 Variation of KE^+ with $\tilde{u}^+$ in the inner layer.[7]

equation

$$KE^+ = 2.25\tilde{u}^+ - 1.3 \quad (5)$$

Equation (5) is valid for the developing and fully developed boundary layer in smooth channel flow and over a range of Reynolds numbers.

The application of Eq. (3) to the inner layer is illustrated in Fig. 4, which is based on Clark's data. The stright line shown in the figure is Eq. (5), which is seen to fit the data reasonably well in the inner region up to the lowest value of y^+ investigated, $y^+ = 5$.

Conclusions

It is found that the linear relationship ($-\overline{uv}^+ = A\,KE^+$) is, in general, not accurate when applied to pipe and channel flows. In such flows, however, the turbulent kinetic energy varies linearly with the axial component of turbulence intensity. The linearity holds for rough and smooth ducts, for the inner and outer layers, and over a range of Reynolds numbers. The data analyzed in the present study indicate the following relationships: 1) for pipe flow, Eq. (4), with $5\times10^4 \le Re \le 5\times10^5$, $0.1 \le y/d \le 1$, $x/d \ge 27$; and 2) for channel flow, Eq. (5), with $1.23\times10^4 \le Re_c \le 2.59\times10^5$, $y^+ \ge 5$, $x/d \ge 40$. Equations (4) and (5) are seen to be in good agreement, and they are believed to be new.

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Technical Comments

Comment on "Some Remarks on the Beck Problem"

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THE critical buckling load for a cantilever column loaded by a tangential follower of force P is given by El Naschie[1] as

$$P_{cr} = 20.19EI/\ell^2 \quad (1)$$

where EI is the bending stiffness and ℓ is the length of the column. Equation (1) represents an instability boundary for the case in which the column structure is considered to be massless with a single concentrated mass at the tip. This result is well known and often cited in the literature[2-4] based on essentially the same dynamic stability analysis as is given in Ref. 1. However, Refs. 2-4 also clearly show that this critical tangential follower load value at which dynamic instability occurs is specific to the mass distribution assumed and cannot be considered to have applicability to columns with other mass distributions.

Thus, the particular value of critical load given by Eq. (1) cannot be considered to be a static property of a tangentially loaded column, and the closeness of the numerical coefficient 20.19 in Eq. (1) to the coefficient 20.05 obtained by Beck for dynamic instability of a tangentially loaded column of uniform mass with no concentrated tip mass must be regarded as not especially significant. In fact, as shown by the work of Pflüger quoted in Refs. 2 and 3, this coefficient varies with the ratio of tip mass to column structural mass, reaching a value near 16 when the ratio of masses is unity.

In general, the critical instability conditions for a tangentially loaded cantilever column are dynamic and cannot be found by static analysis.[2-4] The process which brings about instability is frequency coalescence followed by oscillatory divergent motion. The idealized case of a massless column with tip mass is special, since it constitutes a dynamical system with a single degree of freedom. In this special case, there is only one frequency, so that no frequency coalescence can occur. Bolotin[2] suggests that in this case, instability may be viewed as occurring through coalescence of the one finite frequency with one of the infinite frequencies of the massless column. However, although this conceptual analogy may have some tutorial value, it is far from completely satisfactory since in the ordinary case of frequency coalescence the instability is oscillatory divergent, whereas in the case of a single dynamical degree of freedom the divergence is nonoscillatory. Therefore, the addition of energy to the system by the nonconservative follower forces through cyclic motion, as described for example by Von Kármán and Biot,[5] cannot occur because at least two degrees of freedom are required.

In the case of a massless column with point mass at the tip, the characteristics of the column enter only statically, as a spring constant, whose value is simply the solution of the differential equation for a uniform column loaded by tip force F_t

$$EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = 0 \quad (2)$$

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